Note Title 4/16/2019

$$X(\omega) = \int_0^{\infty} e^{-j\omega t}\, dt$$

$$X(b+j\omega) = \int_0^{\infty} e^{-(b+j\omega)t}\, dt$$

$$= \int_0^{\infty} e^{-bt} e^{-j\omega t}\, dt$$

$$e^{j\omega t} = \cos(\omega t) - j\sin(\omega t) \quad \text{— (1)}$$

$$e^{-bt} e^{-j\omega t} = e^{-bt}(\cos(\omega t) - j\sin(\omega t)) \quad \text{— (2)}$$

$$x(t) = e^{-bt}u(t)$$

$$X(s) = \int_0^{\infty} x(t)\, e^{-st}\, dt$$

$$X(s) = \int_0^{\infty} e^{-bt} e^{-st}\, dt = \int_0^{\infty} e^{-(b+s)t}\, dt$$

$$= -\left(\frac{1}{b+s}\right)\left(e^{-(b+s)t}\right)\Big|_0^{\infty} = \frac{1}{s+b}$$

$$e^{-bt}u(t) \leftrightarrow \frac{1}{s+b} \qquad \sigma > -b \quad s > -b$$

$$u(t) \leftrightarrow \frac{1}{s} \rightarrow \sigma > 0$$

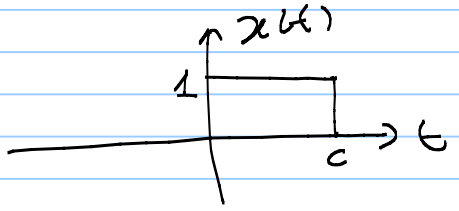
Ex

$$x(t) = u(t) + e^{-t}u(t)$$

$$X(s) = \frac{1}{s} + \frac{1}{s+1} = \frac{(s+1)+s}{s(s+1)} = \frac{2s+1}{s(s+1)}$$

Ex

$$x(t) = \begin{cases} 1, & 0 \le t \le c \\ 0, & \text{all other values of } t \end{cases}$$



$$\boxed{x(t) = u(t) - u(t-c)}$$

$$X(s) = \frac{1}{s} - \frac{1}{s}e^{-cs} = \frac{1-e^{-cs}}{s}$$

Ex

$$x(t) = r(t)$$

$$r(t) = t\,u(t)$$

$$R(s) = (-1)^1 \frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2}$$

$$\boxed{r(t) \longleftrightarrow \frac{1}{s^2}}$$

Ex

$$x(t) = [u(t) - u(t-c)]\,e^{at}$$

$$u(t) - u(t-c) \longleftrightarrow \frac{1-e^{-cs}}{s}$$

$$\therefore X(s) \leftrightarrow \frac{1 - e^{-c(s-a)}}{s-a}$$

Ex

$$x(t) = \cos(\omega t)\, u(t)$$

$$u(t) \leftrightarrow \frac{1}{s}$$

$$X(s) = \frac{1}{2}\left[\frac{1}{s+j\omega} + \frac{1}{s-j\omega}\right] = \frac{1}{2}\left[\frac{s-j\omega+s+j\omega}{(s+j\omega)(s-j\omega)}\right]$$

$$= \frac{1}{2}\,\frac{2s}{s^2+\omega^2} = \frac{s}{s^2+\omega^2}$$

$$\cos(\omega t)\, u(t) \leftrightarrow \frac{s}{s^2+\omega^2}$$

Ex

$$\sin(\omega t)\, u(t) \leftrightarrow \frac{j}{2}\left[\frac{1}{s+j\omega} - \frac{1}{s-j\omega}\right]$$

$$\leftrightarrow \frac{j}{2}\left[\frac{s-j\omega-s-j\omega}{s^2+\omega^2}\right]$$

$$= \frac{j}{2}\left[\frac{-2j\omega}{s^2+\omega^2}\right] = \frac{\omega}{s^2+\omega^2}$$

$$\sin(\omega t)\, u(t) \leftrightarrow \frac{\omega}{s^2+\omega^2}$$

GX

$$2\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} - 4y(t) = \frac{dx(t)}{dt} + 2x(t)$$

Find the Transfer function $\Rightarrow$ I.C. $\equiv 0$

$$2s^2Y(s) + 3sY(s) - 4Y(s) = sX(s) - 2X(s)$$

$$[2s^2 + 3s - 4]Y(s) = [s - 2]X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s-2}{2s^2+3s-4}$$

$$X(s) = \frac{s+2}{s^2+2s+2}, \text{ find the initial value}$$

$$x(0) = \lim_{s\to\infty} sX(s)$$

$$= \lim_{s\to\infty} \frac{s^2+2s}{s^2+3s+2} = \lim_{s\to\infty} \frac{1+\frac{2}{s}}{1+\frac{3}{s}+\frac{2}{s^2}}$$

$$= 1$$